MTH 304: Metric Spaces and Topology

Practice Assignment IV: Countability axioms

1. Reading assignment: Read through the proof of the fact that $\mathbb{R}^\ell$ is Lindelöf from Munkres.

2. Establish assertion 1.12 (ix) from the lesson plan.

3. Prove or disprove (by giving a counterexample, wherever required) the following statements.

   (a) A metrizable space is second countable.

   (b) Every basis of a second countable space contains a countable basis.

   (c) A compact metrizable space is second countable.

   (d) The space $\mathbb{R}_\ell$ is metrizable.

   (e) A metrizable and separable space is second countable.

   (f) A closed subspace of a Lindelöf space is Lindelöf .

   (g) A metrizable and Lindelöf space is second countable.

   (h) A countable product of separable spaces is separable.

   (i) A closed subspace of a separable space is separable.

   (j) Continuous image of a separable space is separable.

   (k) Continuous image of a Lindelöf space is Lindelöf.

   (l) Continuous image of a first (or second) countable space space is first (or second) countable.

   (m) Image under an open continuous map of a first (or second) countable space is first (or second) countable.

   (n) If X is compact and Y is Lindelöf , then $X \times Y$ is Lindelöf.

   (o) In a separable space, every collection disjoint open sets is countable.

4. Which of the four countability axioms does the space $\mathbb{R}^\infty$ with the uniform topology satisfy?